Dept. of Civil & Environmental Engineering

Fracture Mechanics
Term Project

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From fracture tests on concrete specimens of different sizes peak loads have been recorded. The basic material properties known prior to these tests are :

Young's modulus = 4.5×10^6 psi
Tensile strength, f'_t = 450 psi (= f_u)
Maximum aggregate size, d_a = 0.5 in., Unit weight = 145 pcf.

From the given peak loads P_u calculate σ_N for each specimen. Obtain the parameters, B and d_0, of the size effect law from linear regression analysis. Compute the values of fracture energy G_f and the effective length of the fracture process zone c_f. Determine brittleness number β for each size. Plot the size effect law along with the LEFM and strength criteria. Using the values of G_f and c_f , the R-curve can be formulated on the basis of size effect. Compute the load versus load-line deformation response of the three sizes of specimens. Check if the peak loads thus predicted are close th the test data. From the area under the load-deformation curves of each size find G_w, the fracture energy from the RILEM method.

Note : (1) Assume plane stress conditions.
(2) Take $C_n = 1$, $\sigma_N = P_u / bd$

Results should include :

Values for B, d_0, G_f and c_f.
Table showing P_u, σ_N and $(f_u/\sigma_N)^2$ for each specimen.
Table showing predicted peak loads and peak deformations, area under load-deformation curve, β, G_w and G_w/G_f for each size.
Plots of (1) load-deformation diagrams of the three sizes of specimens together, (2) linear regression analysis {X=d, Y=$(f_u/\sigma_N)^2$}, (3) Size effect law along with the strength and LEFM criteria {X=d/d_0, Y= σ_N/Bf_u both axes logarithmic} and (4) G_w/G_f versus specimen size, d.

Relevant Equations

Size Effect Law :

$$\sigma_N = c_n \left(\frac{E' G_f}{g'(\alpha_0) c_f + g(\alpha_0) d} \right)^{1/2} = \frac{B f_u}{\sqrt{1+\beta}}, \quad \beta = \frac{d}{d_0}$$

E' = E for plane stress and E' = E/(1-ν^2) for plane strain.

Linear Regression :

$$Y = AX + C, \; X = d, \; Y = (f_u/\sigma_N)^2, \; B = 1/\sqrt{C}, \text{ and } d_0 = C/A.$$

$$G_f = \frac{B^2 f_u^2}{c_n^2 E'} d_0 g(\alpha_0), \qquad c_f = \frac{d_0 g(\alpha_0)}{g'(\alpha_0)}, \qquad f(\alpha) = \frac{K_I b \sqrt{d}}{P}$$

$$g(\alpha) = \{ f(\alpha)^2 \}, \qquad g'(\alpha) = \partial g(\alpha)/\partial \alpha.$$

R-curve :

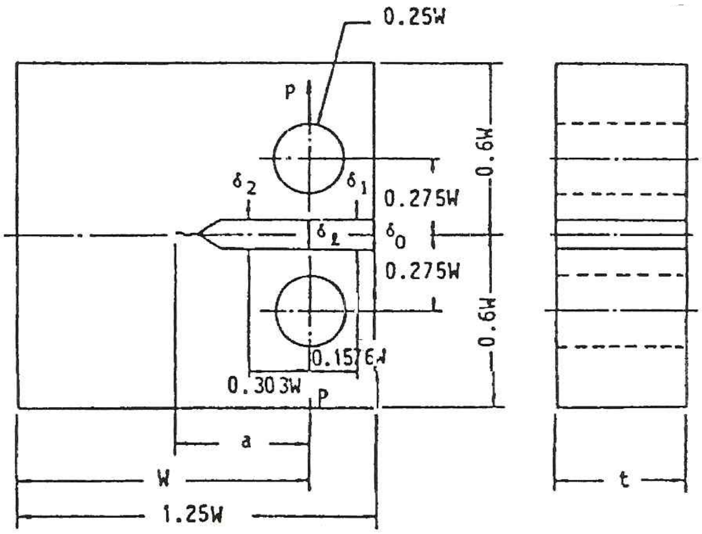
$$R(c) = G_f \frac{g'(\alpha)}{g'(\alpha_0)} \frac{c}{c_{f,}} \qquad \frac{c}{c_f} = \frac{g'(\alpha_0)}{g(\alpha_0)} \left(\frac{g(\alpha)}{g'(\alpha)} - \alpha + \alpha_0 \right)$$

Test Data

Specimen	Data to be analyzed by	Size, d (in)	Peak Loads, P_u (lbs)
3-point bend d = b, b = t, α = A, s/b = 4, α_0 = 0.4, b = 1.5 in	Tuguldur 정한윤	1.5	125, 129, 130
		3.0	208, 205, 210
		6.0	321, 317, 320
	이선민	2.0	155, 159, 160
		4.0	245, 249, 249
		8.0	377, 380, 381
	이후범 Mohammad	5.0	285, 287, 230
		10.0	430, 433, 431
		20.0	630, 625, 629
Compact Tension d = W, b = t, α_0 = 0.3, b = 1.5 in	이호재	2.0	220, 225, 223
		4.0	356, 352, 355
		8.0	535, 540, 539
	Syed 박지혁	2.5	260, 261, 265
		5.0	405, 408, 409
		10.0	608, 610, 611
	이정운	3.0	296, 294, 296
		6.0	455, 453, 459
		12.0	672, 680, 673

Compact Tension Specimen (CT)

- From "Stress Intensity Factors Handbook", Ed, Y. Murakami (1987)



(1) Stress Intensity Factor

[Reference] J.E. Srawley [1], [2]

[Method] Based on boundary collocation results

[Accuracy] ±0.5% for 0.2 ≤ a/w ≤ 1.0

$$K_I = \frac{P}{tw^{1/2}} f_I(\alpha), \quad \alpha = \frac{a}{W}$$

$$f_I(\alpha) = \frac{(2+\alpha)(0.886+4.64\alpha-13.32\alpha^2+14.72\alpha^3-5.6\alpha^4)}{(1-\alpha)^{3/2}}$$

(2) Displacements

Crack opening displacement at load line

[Reference] A.Saxena and S.J.Hudak, Jr. [3]

[Method] Neman's modified boundary collocation techniques

[Accuracy] ±0.5% for 0.2 ≤ a/w ≤ 0.975

$$\delta_\ell = \frac{P}{E't} V_\ell(\alpha), \qquad \alpha = \frac{a}{W}$$

$$V_\ell(\alpha) = \left(\frac{1+\alpha}{1-\alpha}\right)^2 (2.1630+12.219\alpha-20.065\alpha^2-0.9925\alpha^3+20.609\alpha^4-9.9314\alpha^5)$$

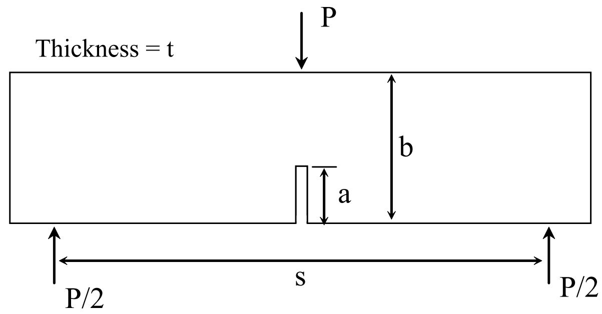
THREE-POINT BENDING SPECIMEN

- From "The stress Analysis of Cracks Handbook" Tada, Paris & Irwin (1985)

(1) Stress Intensity Factor

$$\sigma = \frac{6M}{tb^2} \quad (M = \frac{Ps}{4})$$

$$K_I = \sigma\sqrt{\pi a}\,F(a/b)$$



For s/b = 4, the following approximate formula is valid for any a/b =A within 0.5%

$$F(a/b) = \frac{1}{\sqrt{\pi}} * \frac{1.99 - A(1-A)(2.15 - 3.93A + 2.7A^{2)}}{(1+2A)(1-A)^{3/2}}$$

Reference : Srawley 1976

(2) Additional Load Point Displacement due to Crack

$$\Delta_{crack} = \Delta$$

following formula has better than 1% accuracy for and a/b.

s/b = 4 :

$$V_2(a/b) = \left(\frac{a/b}{1-a/b}\right)^2 \{5.58 - 19.57(a/b) + 36.82(a/b)^2 - 34.94(a/b)^3 + 12.77(a/b)^4\}$$

Method : Paris's equation (Paris 1957) (See Appendix B)

Reference : Tada 1973